SEBHA UNIVERSITY JOURNAL OF PURE & APPLIED SCIENCES VOL.24 NO. 3 2025
DOI: 10.51984/JOPAS.V24I3.4315

مجلة جامعة سبها للعلوم البحتة والتطبيقية
Sebha University Journal of Pure & Applied Sciences

Journal homepage: www.sebhau.edu.ly/journal/jopas

Assessment of Upper Limb Muscle Exertion in School-Aged Children during Load Carrying

Faisal Abdalla[a], Ahmed Elsakori[b], Hafed Saleh[c]*

[a]Department of Mathematics , The Libyan Academy-Benghazi, Benghazi, Libya.
[b]Department of Mathematics, Faculty of Arts and Sciences-Al marj, University of Benghazi, Benghazi, Libya.
[c]Department of Mathematics, College of Electrical and Electronics Technology, Benghazi, Libya.

Keywords:
Backpack.
Load Carriage.
Muscle Forces.
Safe Load.
Optimization.

ABSTRACT

Carrying heavy backpacks exceeding 10% of body weight (BW) is a widespread issue among primary school students in developing countries, often linked to musculoskeletal injuries. This study proposes redistributing excess backpack loads to the arms to reduce spinal strain and evaluates the safety of this approach. An optimisation model was developed to estimate arm muscle forces in ten healthy students (age: 8.09 ± 0.85 years) during treadmill walking under three load conditions, which were 0-L (no arm load), 1-L (2.2 kg arm load), and 2-L (4.5 kg). Kinematic data were captured using a Vicon motion analysis system, and muscle forces derived from the model were compared to their maximum forces (Fmax). Results demonstrated that muscle forces increased significantly with load magnitude: biceps and triceps forces remained below safe load limits in 0-L, with 113.46 N and 115.06 N, respectively, and 1-L, with 228.34 N and 231.12 N. However, at 2-L, triceps force was 309 N,which exceeded its maximum force (Fmax = 280 N). Paired t-tests confirmed statistically significant differences between all conditions. These findings suggest that transferring 2.2 kg (1-L) from the back to the arms is biomechanically safe, whereas 4.5 kg (2-L) poses a risk of overloading arm muscles. This study provides actionable insights for optimising load distribution in schoolchildren to prevent spinal injury.

تقييم جهد عضلات الطرف العلوي لدى الأطفال في سن المدرسة أثناء حمل الأثقال

فيصل عبدالله[a]، أحمد السكوري[b]، حافظ صالح[c]*

[a]قسم الرياضيات، الأكاديمية الليبية، بنغازي، ليبيا.
[b]قسم الرياضيات، كلية الاداب والعلوم – المرج، جامعة بنغازي، ليبيا.
[c]قسم الرياضيات، كلية التقنية الكهربائية والالكترونية، بنغازي، ليبيا.

الكلمات المفتاحية:
حقيبة الظهر.
حمل الثقل.
قوى العضلات.
الحمل الآمن.
التحسين.

الملخص

يُعد حمل حقائب الظهر الثقيلة التي تتجاوز 10% من وزن الجسم مشكلة شائعة بين طلاب المدارس الابتدائية في البلدان النامية، وغالبًا ما يرتبط بإصابات الجهاز العضلي الهيكلي. تقترح هذه الدراسة إعادة توزيع أحمال حقائب الظهر الزائدة على الذراعين لتخفيف إجهاد العمود الفقري. طُوّر نموذج أستمثالا (تحسين) لتقدير قوى عضلات الذراع لدى عشرة طلاب أصحاء (أعمارهم: 8.09 ± 0.85 سنة) أثناء المشي على جهاز المشي في ظل ثلاثة ظروف حمل: L-0 (بدون حمل على الذراع)، وL-1 (2.2 كجم)، وL-2 (4.5 كجم). تم الحصول على البيانات الحركية باستخدام نظام تحليل الحركة Vicon، وقورنت قوى العضلات المُستقاة من النموذج بالحد الأقصى لقوتها. أظهرت النتائج أن قوى العضلات تُقدَّر بشكل ملحوظ مع شدة الحمل: إذ ظلت قوى العضلة ذات الرأسين والعضلة ثلاثية الرؤوس أقل من الحد الآمن للحمل في حالتي L-0 (113.46 نيوتن و115.06 نيوتن ، على التوالي) وL-1 (228.34 نيوتن و231.12 نيوتن). ومع ذلك، في حالة L-2، تجاوزت قوى العضلة ثلاثية الرؤوس أقصى قوى لها (309 نيوتن مقابل Fmax = 280 نيوتن). أكدت اختبارات

*Corresponding author.
E-mail addresses: faisalse@lab.edu.ly, (A. Elsakori) ahmed.elsakori@uob.edu.ly, (H. Saleh) hafed_saleh@ceet.edu.ly.
Article History : Received 01 September 25 - Received in revised form 23 November 25 - Accepted 01 December 25

t المزدوجة وجود فروق ذات دلالة إحصائية بين جميع الحالات (p < 0.005). تشير هذه النتائج إلى أن نقل 2.2 كجم (1-
L) من الظهر إلى الذراعين آمن من الناحية البيوميكانيكية، بينما يُشكل نقل 4.5 كجم (L-2) خطر زيادة تحميل عضلات
الذراع. تُقدم هذه الدراسة رؤى عملية لتحسين توزيع الحمل لدى أطفال المدارس للوقاية من إصابات العمود الفقري.

1. Introduction

Carrying school backpacks loaded with textbooks has long been a persistent issue for students, with various medical problems reported as a result of heavy backpack use Efforts have been made to reduce backpack loads, and the impact of different loads on primary school students has been widely studied. Nevertheless, in many developing countries, students are still reported to carry backpacks weighing more than 10% of their BW [1–3]. While many studies have investigated the effects of carrying a backpack [4–6], researchers have provided few practical recommendations beyond simply reducing the weight. Carrying heavy backpacks can have lasting effects throughout a student's life and is particularly damaging to students at the elementary school level [7]. Various researchers have suggested specific load limits and proposed strategies to minimise the negative impacts of heavy backpack carriage [8]. In particular, it is commonly recommended that schoolchildren should not carry more than 10% of their BW [5, 9]. In this study, we explore the idea of transferring part of the backpack load to the arms of elementary school students.

This approach requires analysing the kinetics and kinematics of the students' arms to understand the behaviour of arm muscles during load carriage and determine how much load can safely be transferred. Previous researchers have investigated arm muscle dynamics using experimental and analytical techniques [10, 11]. Estimating individual muscle forces typically involves optimisation methods to solve the redundant system [27, 28]. Dynamic optimisation methods have been widely employed to calculate muscle forces by minimising an objective function over time [12]. Such problems can be addressed using optimal control, static optimisation, or calculus of variations (COV) methods [12–14]. COV, in particular, is an analytical approach that derives optimal solutions through the Euler–Lagrange differential equation [15–17]. Molinder [18] demonstrated the application of the COV method in solving dynamic optimisation problems within the field of economics, relying on the Euler–Lagrange equation to find optimal solutions.

Motion capture data are essential for simulating and solving the optimisation problem to estimate muscle forces accurately. The objectives of this study are to estimate the forces generated by arm muscles using optimisation techniques and to determine a safe load that can be carried by students' arms. It is hypothesised that all subjects had sufficient rest before the experiments began.

2. Subjects

Ten primary school students (6 males and 4 females) participated in this study to investigate their arm muscles during walking with three different load conditions. The subjects were healthy and had no history of muscle pain with age mean (±SD) 8.09 (±0.85) years, height 127.2 (±9.15) cm, and weight 25.54 (±4.52) kg as well. All subjects performed the activity carrying a backpack and walking on a treadmill with self-selected speed of 2.3 (±0.3) km/m.

3. Experiment Design

External loads were applied to the distal segment at the center of mass (COM). The arm model was investigated under dynamic conditions with three different load conditions, 0-L as zero load, 1-L is the subtraction of 10% of the BW and the maximum load, 4.8 kg or 47.04 N, which was obtained from [2]. Finally, the third load condition (2-L) was identified as the double of the 1-L. Motion capture data were collected from the subjects for 20 seconds using a four-camera motion analysis system (Vicon Motion System, 1.5.2), processed at 50 Hz and derived using the plug-in-gait model. The data were analyzed using MATLAB (7.11) for filtering using the smoothing spline filter and then numerically derived to calculate the angular velocity and accelerations of the respective joints to be included in the dynamic model.

4. Arm Biomechanical Model

The right arm was modelled in a two-dimensional plane, with two degrees of freedom (DOFs), which consists of two segments: upper arm and combined forearm-hand, as one segment, linked by elbow joint actuated by muscles to cause flexion/extension movements (figure 1). The segments were considered to be rigid bodies, with lengths l_1 and l_2 and the elbow joint was assumed to be hinge and frictionless. Local coordinate systems (LCS) for each segment were assumed to be attached to the arm segments and located at the rotation centers at the joints centers of origin points. Each segment has its LCS, where $O_1X_1Y_1$ is the LCS at the shoulder joint as unmovable coordinate system and the $O_2X_2Y_2$ is the LCS attached to the center of the elbow joint. The shoulder and elbow joint angles, $\emptyset_1$ and $\emptyset_2$, are the flexion/extension angles of the respective joints, as in figure 1. Forces of gravity on the arm segments, $G_1$and G_2, are the weight forces of the upper arm and forearm-hand segments, and they affect at the center of masses (COMs). The point of action of the gravitational forces on the arm segments was determined as the product of the segment length and the proportion as the distance between the proximal end of the respective segment and the location of its gravity force. The proportion values for the upper arm segment, $r_1 = 0.436$, and the forearm-hand segment, $r_2 = 0.682$, were taken from [20].

Five arm muscles were selected for inclusion in the biomechanical model of the subjects' right arm, the biceps, triceps, brachialis, flexor carpi radials (FCR), and extensor digitrum (ED) muscles to be characterized by the force actions, F_i, as one force. Forces caused by ligaments are ignored compared with muscle forces. The moment arm and inclination angle for each muscle are symbolized by r_{ij} and θ_{ij}, respectively, where i represents a particular muscle and j represents a respective joint. The inclination angle of the muscle is defined as the angle between the line of muscle action and the segment that is actuated by this muscle. The moment arms and physiological cross-sectional areas (*PCSAs)* of muscle forces and muscle inclination angles are obtained from Raikova [21], Veeger et al. [22], and an [23].

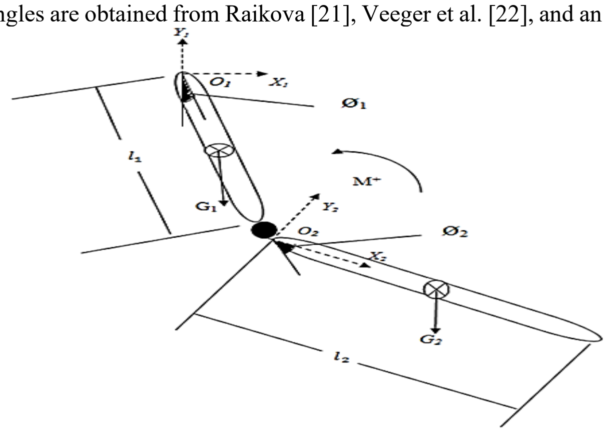

Fig. 1: Free-body diagram of the arm model in the sagittal plane.

The weights $G_1$and G_2 exert clockwise moments on the upper arm and forearm-hand segments about the shoulder and elbow joints, respectively. The investigated muscle forces have two different actions that move the segments of the biomechanical model, upward in a counterclockwise direction and extend in the clockwise direction. The equations of motion were formulated with respect to the LCSs and mathematically describe the arm segments in dynamic pattern using Newton's laws with three different load conditions. The equality constraints are represented by equilibrium moment equation for each DOF. All muscle forces are limited to be $0 \leq F_i \leq F_{max}$. The maximum muscle forces were calculated by the product of the constant maximum stress, which was assumed to be 62 N/cm^2 [24], and the *PCSA* for each muscle. The masses and the moments of inertias of the arm segments were determined based on the subjects' body masses.

5. The Mathematical Formulation of the Dynamic Model

According to Newton's laws, the equations of motion represent mathematically the dynamic pattern of the arm segments [29,30]. The equations (1) characterize the upper arm segment during walking, where equations (1a) and (1b) represent the total sum of the horizontal and vertical forces acting on the upper arm segment. Equation (1c) is the moment equilibrium equation that represents the total sum of the moments of all forces acting on the upper arm segment.

$$\cos(\theta_{11})\,F_1 + \cos(\theta_{12})\,F_2 - F_{Sx} + \sin(\varphi_1+\varphi_2)\,F_{Ex} + \cos(\varphi_1+\varphi_2)\,F_{Ey} = m_1 a_{1x}. \tag{1a}$$

$$\sin(\theta_{21})\,F_1 + \sin(\theta_{22})\,F_2 - F_{Sy} - \cos(\varphi_1+\varphi_2)\,F_{Ex} + \sin(\varphi_1+\varphi_2)\,F_{Ey} = m_1 a_{1y} \tag{1b}$$

$$r_{11}\,F_1 + r_{12}\,F_2 + r_{e1}\,F_{Ex} + r_{e2}\,F_{Ey} - r_{g1}\,F_{G1} - M_{ext1} = I_1\alpha_1. \tag{1c}$$

$$r_{e1} = l_1 \sin(\varphi_2). \tag{1d}$$

$$r_{e2} = l_1 \cos(\varphi_2). \tag{1e}$$

$$r_{g1} = r_1 \sin(\varphi_1). \tag{1f}$$

Equations (2a) and (2b) are the horizontal and vertical forces acting on forearm-hand segment, and (2c) is the moment equation in a dynamic state, which is the total sum of the moments of all forces acting on the forearm-hand segment.

$$-\sin(\theta_{21})\,F_1 - \sin(\theta_{22})\,F_2 - \sin(\theta_{23})\,F_3 - \sin(\theta_{24})\,F_4 - \sin(\theta_{25})\,F_5 - F_{Ex} + F_{G2}\cos(\varphi_1+\varphi_2) = m_2 a_{2x} \tag{2a}$$

$$\cos(\theta_{21})\,F_1 + \cos(\theta_{22})\,F_2 + \cos(\theta_{23})\,F_3 + \cos(\theta_{24})\,F_4 + \cos(\theta_{25})\,F_5 - F_{Ey} - F_{G2}\sin(\varphi_1 + \varphi_2) = m_2 a_{2y} \tag{2b}$$

$$r_{21}F_1 + r_{22}F_2 + r_{23}F_3 + r_{24}F_4 + r_{25}F_5 - r_{g_2}(F_{G2} + F_{ext}(k)) - M_{ext2} = I_2\alpha_2 \tag{2c}$$

$$g_2 = r_2 \sin(\varphi_1 + \varphi_2). \tag{2d}$$

where m_1 and m_2 are the mass segments, I_1 and I_2 are the moments of inertia of the model segments, α_1 and α_2 are the angular accelerations, a_{1x} and a_{2x} are the x-components of linear acceleration, and a_{1y} and a_{2y} are the y-components of the linear acceleration segments. The lower script '1' indicates the upper arm segment and '2' indicates the forearm-hand segment.

6. Dynamic Optimization Problem

The objective function J of muscle stresses is a quadratic function and constrained by moment equation for each DOF (Equations (3)). The objective function points out how much individual muscle contributes to the activity to be minimized in the optimization task and it is formulated as follows:

$$J = min \int_{t_1}^{t_2} f[F_1, \dot{F_1}, \ldots, F_5, \dot{F_5}, t]dt \tag{3a}$$

such that $f = \sum_{i=1}^{5} \frac{F_i^2}{PCSA_i}$. subject to

$$Q_1 = r_{11}F_1 + r_{12}F_2 - r_{g1}F_{G1} - M_{ext1} - I_1\alpha_1 \tag{3b}$$

$$Q_2 = r_{21}F_1 + r_{22}F_2 + r_{23}F_3 + r_{24}F_4 + r_{25}F_5 - r_{g2}\big(F_{G2} + F_{ext}(k)\big) - M_{ext2} - I_2\alpha_2. \tag{3c}$$

$$0 \le F_i \le F_{maxi} \tag{3d}$$

The Lagrangian equation can be formulated as follows:

$$L = \Big(\frac{F_1^2}{PCSA_1} + \frac{F_2^2}{PCSA_2} + \frac{F_3^2}{PCSA_3} + \frac{F_4^2}{PCSA_4} + \frac{F_5^2}{PCSA_5}\Big) + \sum_{k=1}^{2} \lambda_k\,(t) Q_k. \tag{4}$$

Thus, the Euler–Lagrange equation (5) can be obtained by deriving Equation (4) with respect to the design variables and the Lagrange multipliers (λ_1 and λ_2) [29, 30].

$$\frac{\partial L}{\partial F_i} - \frac{d}{dt}\Big(\frac{\partial L}{\partial \dot{F_i}}\Big) + \lambda_1 \frac{\partial Q_1}{\partial F_i} + \lambda_2 \frac{\partial Q_2}{\partial F_i} = 0. \tag{5}$$

The joint reaction forces ($JRFs$) would not be taken into account in the optimization task, which can be calculated subsequent to the estimation of the unknown muscle forces.

$$2\frac{F_1}{PCSA_1} + \lambda_1 r_{11} + \lambda_2 r_{21} = 0. \tag{6a}$$

$$2\frac{F_2}{PCSA_2} + \lambda_1 r_{12} + \lambda_2 r_{22} = 0. \tag{6b}$$

$$2\frac{F_3}{PCSA_3} + \lambda_2 r_{23} = 0. \tag{6c}$$

$$2\frac{F_4}{PCSA_4} + \lambda_2 r_{24} = 0. \tag{6d}$$

$$2\frac{F_5}{PCSA_5} + \lambda_2 r_{25} = 0. \tag{6e}$$

$$r_{11}F_1 + r_{12}F_2 = r_{g1}F_{G1} + M_{ext1} + I_1\alpha_1 \tag{6f}$$

$$r_{21}F_1 + r_{22}F_2 + r_{23}F_3 + r_{24}F_4 + r_{25}F_5 = r_{g2}\big(F_{G2} + F_{ext}(k)\big) + M_{ext2} + I_2\alpha_2. \tag{6g}$$

The equations (3b and 3c) as well as the derived Lagrangian equation (5) make together a determinate system (6) that can be readily solved, using the equation $X = A^{-1} \cdot B$, according to the principle of COV. The dynamic optimization model was simulated using MATLAB to estimate the right arm muscle forces during walking throughout the three load conditions. Collected kinematic data were also exported to MATLAB code to be used as input in the simulation for estimating the muscle forces in the dynamic condition.

7. Results

The dynamic optimization method was used to estimate the arm muscle forces during walking with three load conditions to investigate the muscle patterns of the students' right arms throughout the identified load conditions, 0-L, 1-L, and 2-L. The muscle forces were estimated using the dynamic optimization model that was analytically solved using COV.

A comparison was made for the contribution of the right arm muscles for each load condition. As shown in Figures 2, 3, and 4 throughout the estimated muscle forces for every load condition, each muscle force was influenced by the change in the applied load. The muscle forces estimated by the model were represented by root mean square (RMS) values, which were calculated and compared to evaluate the level of activity. The biceps and triceps muscles had the largest estimated muscle forces during walking where they exerted forces with RMS of 113.46 N and 115.06 N, respectively, during 0-L condition with root mean square difference (RMSD) of 2.18 N. During 1-L condition, the results of biceps and triceps muscles also had the largest forces among the arm muscles, with RMS of 228.34 N and 231.12 N, respectively, and 2.92 N as RMSD. The RMS values were also calculated throughout 2-L condition, and it was found that biceps and triceps muscles exerted 308.31 N and 309 N as RMS with RMSD 1.43 N. Throughout the three load conditions, FCR and ED muscles had the smallest estimated forces where FCR muscle exerted forces of 18.76, 38.16, and 55.97 N, whereas ED exerted forces of 22.93 N, 46.64 N, and 68.41 N as RMS values. The RMSDs were calculated, and it was found that 4.17 N, 8.48 N, and 12.44 N represent the RMSD values between FCR and ED muscles for 0-L, 1-L, and 2-L conditions, respectively. As one of the investigated arm muscle, brachialis muscle exerted forces in between biceps and triceps muscle forces as largest forces and FCR and ED muscle forces as the smallest muscle forces for all load conditions. Brachialis muscle forces had 70.88, 144.15, and 211.45 N as RMS values during 0-L, 1-L, and 2-L conditions, respectively, and averaged RMSD values were calculated between biceps and FCR muscle forces that were found to be 77.25 and 181.01 N, respectively.

Within the load conditions, the activity level of each arm muscle was evaluated and compared with each other to show the change in muscle activity with different loads applied on the arm throughout walking. The results showed that the muscle forces were always greater during 2-L condition than other conditions and contributions to the total muscle loading were less during 0-L condition than 1-L and 2-L conditions.

The estimated muscle forces with 0-L and 1-L conditions were compared with their maximum forces to be found less than their maximum forces and triceps muscle exceeded its maximum muscle force with 2-L condition

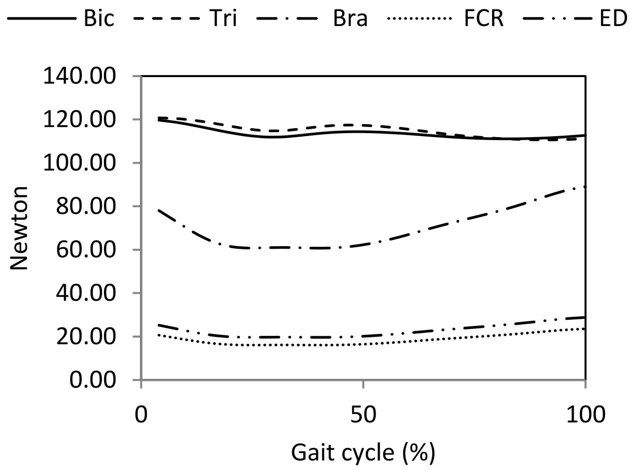

Fig. 2: Biceps, triceps, brachialis, FCR, and ED muscles forces estimated by the dynamic model for a complete gait cycle with 0-L.

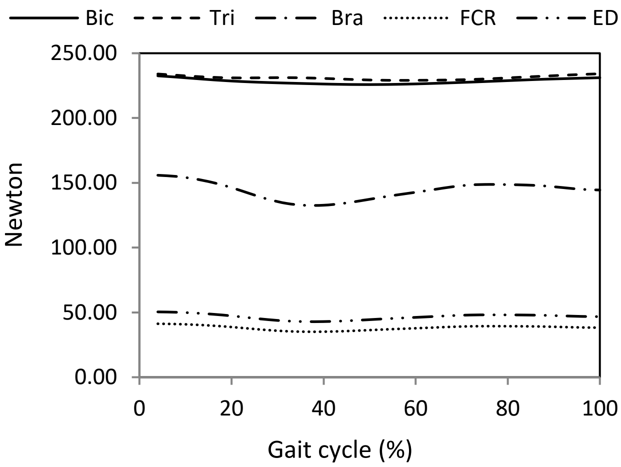

Fig. 3: Biceps, triceps, brachialis, FCR, and ED muscles forces estimated by the dynamic model for a complete gait cycle with 1-L.

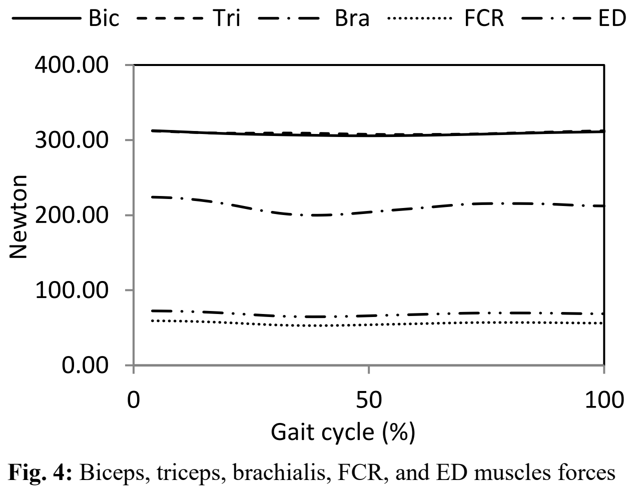

Fig. 4: Biceps, triceps, brachialis, FCR, and ED muscles forces estimated by the dynamic model for a complete gait cycle with 2-L.

The calculation of the *JRFs* was made to demonstrate the influence of the change in load on the contact forces of the respective joints and compared with each other within load conditions. The *JRFs* of shoulder and elbow joints were plotted, as shown in Figures 5 and 6, and it was found for each load condition that *JRFs* of shoulder were larger than *JRFs* of elbow, and they were also influenced by the change in load. Maximum *JRFs* occurred in the respective joints with 2-L condition, whereas those joints had the smallest values with 0-L condition.

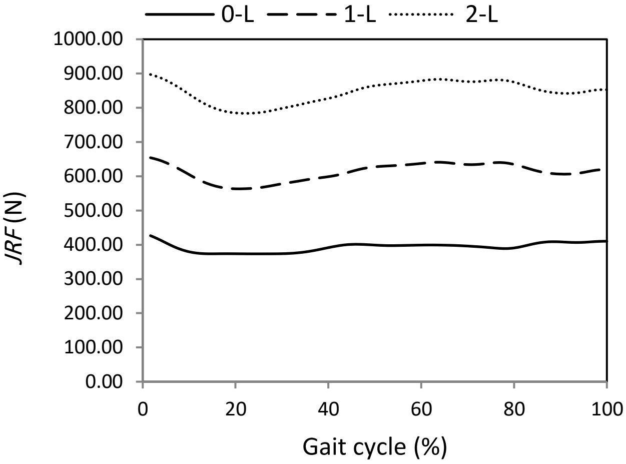

Fig. 5: Shoulder *JRFs* for three load conditions, 0-L, 1-L, and 2-L, during a complete gait cycle.

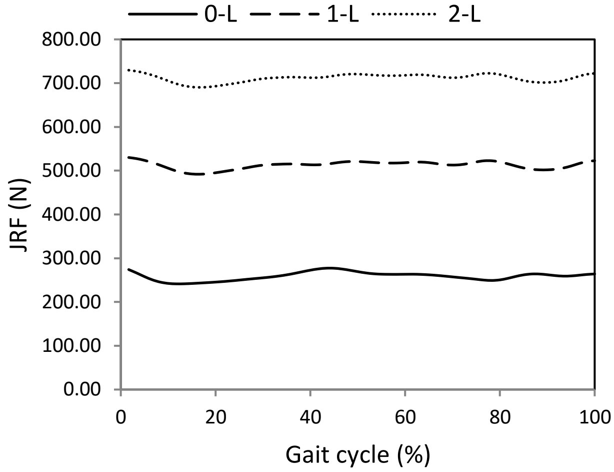

Fig. 6: Elbow *JRFs* for three load conditions, 0-L, 1-L, and 2-L, during a complete gait cycle.

8. Discussion

The aim of this study is mainly to reduce the backpack weight carried by students in basic education by transferring a particular load of the backpack to the arm as an external load applied at the forearm-hand segment. Therefore, the arm muscles were investigated during load carriage throughout the identified load conditions, 0-L, 1-L, and 2-L, for a complete gait cycle. Moreover, the subjects' arm muscle forces were estimated and compared with the collected EMG data during walking with the load conditions, and a safe load limit that can be carried by the students was discussed as well.

Biceps and triceps muscle forces had the largest values throughout the load conditions, which indicate that biceps and triceps work as the main actuators of the arm when carrying the load. The results of the FCR and ED muscles, in the role of synergist, indicate that they had no proper contribution to the total loading of these muscles. Brachialis works as a synergist muscle; its contribution during the activity is clearly noticed and it is more active than the other synergist muscles, but less active than the biceps and triceps muscles.

According to the large contribution of the total load, the results showed that the two-joint muscles, biceps and triceps, work as the main movers while the brachialis, FCR, and ED muscles function as synergists. Moreover, the flexor muscles investigated in this study—biceps, brachialis, and FCR—were active throughout the load conditions such that they flex the arm upward; and it was found that the extensor muscles—triceps and ED—are also active during flexion activity. The extensor muscles do not necessarily work to oppose motion but may provide stability and stiffness to a joint assuming the role of synergists, which help agonistic muscles perform a desired action [25, 26].

JRFs at the shoulder joint had larger forces compared with the elbow joint, which had less *JRFs* among the load conditions. According to the results of the dynamic optimization model, 1-L may be considered as a safe load to be carried in the arms of children during going and coming back from school as a way to distribute and reassign the heavy load of the students' backpacks. Furthermore, the 2-L condition should be avoided as load applied on a student's arm because biceps and triceps muscle forces estimated by the models exceeded their maximum forces with this load condition.

The increase in external applied load influences the muscle forces, but of course, more effort should be exerted by the student to carry a heavier load. The high values of the muscle forces estimated by the optimization model with 2-L condition refer to the increase in the external applied load, so a safe load should be recommended to be carried by the arms.

9. Conclusion

The dynamic optimisation model successfully demonstrated a substantial dependence of both muscle forces and joint reaction forces (JRFs) on the magnitude of the external load applied to the arm. The model provides clear, actionable safety limits: the 1-L condition is characterised as relatively safe, while the 2-L condition results in an unacceptable risk profile, with predicted forces exceeding validated biomechanical tolerance limits. The statistically significant differences observed between these load conditions underscore the sensitivity of the arm's musculoskeletal system to even moderate increases in weight. In conclusion, these results validate the transfer of a 2.2 kg (1-L condition) load to the arm but provide a robust basis for strongly cautioning against using the arm segment to carry an excess load of 4.5 kg (2-L condition).

10. References

[1] Lasota, A. 2014. Schoolbag weight carriage by primary school pupils. *Work: A Journal of Prevention, Assessment and Rehabilitation* 48 (1):26-12.

[2] Fazrolrozi, & Rambely, A. 2008. Preliminary study of load carriage on primary school children in Malaysia. Paper read at ISBS Conference, Seoul, Korea.

[3] Li, J. X., Hong, Y., &Robinson, P. D. 2003. The effect of load carriage on movement kinematics and respiratory parameters in children during walking. *European Journal of Applied Physiology* 90 (1-2):35-43.

[4] Chansirinukor, W., Wilson, D., Grimmer, K. & Dansie, B. 2001. Effects of backpacks on students: measurement of cervical and shoulder posture. *Australian Journal of physiotherapy* 47 (2):110-116.

[5] Hong, Y., Li, J. X., Wong, A. S. K., &Robinson, P. D. 2000. Effects of load carriage on heart rate, blood pressure and energy expenditure in children. *Ergonomics* 43 (6):717-727.

[6] Dianat, I., Sorkhi, N., Pourhossein, A., Alipour, A., &Asghari-Jafarabadi, M. 2014. Neck, shoulder and low back pain in secondary schoolchildren in relation to schoolbag carriage: Should the recommended weight limits be gender-specific? *Applied Ergonomics* 45 (3):437-442.

[7] Kistner, F., Fiebert, I., &Roach, K. 2012. Effect of backpack load carriage on cervical posture in primary schoolchildren. *Work: A Journal of Prevention, Assessment and Rehabilitation* 41 (1):99-108.

[8] Knapik, J, Harman, E, &Reynolds, K. 1996. Load carriage using packs: A review of physiological, biomechanical and medical aspects. *Applied Ergonomics* 27 (3):207-216.

[9] Ramadan, M. Z., & Al-Shayea, A. M. 2013. A modified backpack design for male school children. *International Journal of Industrial Ergonomics* 43 (5):462-471.

[10] Heller, M. O., Bergmann, G., Deuretzbacher, G., Dürselen, L., Pohl, M., Claes, L., Haas, N. P., &Duda, G. N. 2001. Musculo-skeletal loading conditions at the hip during walking and stair climbing. *Journal of Biomechanics* 34 (7):883-893.

[11] Herzog, W, & Binding, P. 1993. Cocontraction of pairs of antagonistic muscles: analytical solution for planar static nonlinear optimization approaches. *Mathematical Biosciences* 118 (1):83-95.

[12] Davy, D. T., & Audu, M. L. 1987. A dynamic optimization technique for predicting muscle forces in the swing phase of gait. *Journal of biomechanics* 20 (2):187-201.

[13] Yamaguchi, G. T., & Zajac, F. E. 1990. Restoring unassisted natural gait to paraplegics via functional neuromuscular stimulation: a computer simulation study. *Biomedical Engineering, IEEE Transactions on* 37 (9):886-902.

[14] Neptune, R. R., & Hull, M. L. 1999. A theoretical analysis of preferred pedaling rate selection in endurance cycling. *Journal of Biomechanics* 32 (4):409-415.

[15] Troutman, John L. 1995. *Variational calculus and optimal control: optimization with elementary convexity*: Springer-Verlag New York, Inc.

[16] Akhiezer, N. I. 1962. *The calculus of variations*. New York & London: Blaisdell.

[17] Pars, L. A. 2013. *An introduction to the calculus of variations*: Courier Dover Publications.

[18] Molinder, A. 1995. Application of calculus of variations to a continuous time aggregate production model. *International Journal of Production Economics* 41 (1):273-280.

[19] Grubbström, R. W., & Ashcroft, S. H. 1991. Application of the calculus of variations to financing alternatives. *Omega* 19 (4):305-316.

[20] Plagenhoef, S., & Curtis, D. 1971. *Patterns of human motion: A cinematographic analysis*: Prentice-Hall Englewood Cliffs, NJ.

[21] Raikova, R. 2000. Prediction of individual muscle forces using Lagrange multipliers method—a model of the upper human limb in the sagittal plane: I. Theoretical considerations. *Comput Methods Biomech Biomed Engin.* 3 (2):95-107.

[22] Veeger, H, Van der Helm, F, Van Der Woude, L. H. V., Pronk, G. M., & Rozendal, R. H. 1991. Inertia and muscle contraction parameters for musculoskeletal modelling of the shoulder mechanism. *Journal of Biomechanics* 24 (7):615-629.

[23] An, KN, Hui, F. C, Morrey, B. F, Linscheid, R. L, & Chao, E. Y. 1981. Muscles across the elbow joint: a biomechanical analysis. *Journal of Biomechanics* 14 (10):659-669.

[24] Buchanan, T. S. 1995. Evidence that maximum muscle stress is not a constant: differences in specific tension in elbow flexors and extensors. *Medical Engineering & Physics* 17 (7):529-536.

[25] Ait-Haddou, R., Binding, P., & Herzog, W. 2000. Theoretical considerations on cocontraction of sets of agonistic and antagonistic muscles. *Journal of Biomechanics* 33 (9):1105-1111.

[26] Watkins, James. 1999. *Structure and function of the musculoskeletal system*: Human Kinetics Champaign.

[27] Anderson, Frank C, & Pandy, Marcus G. 2001. Static and dynamic optimization solutions for gait are practically equivalent. *Journal of biomechanics* 34 (2):153-161.

[28] Lin, Y.-C., Dorn, T. W., Schache, A. G., &Pandy, M. G. 2012. Comparison of different methods for estimating muscle forces in human movement. Paper read at Proc. Inst. Mech. Eng. H.

[29] Bertsekas, D. P. 1996. *Constrained Optimization and Lagrange Multiplier Methods*. Belmont, MA: Athena Scientific.

[30] Grabowska, K., Grabowski, J. 2008. Variational calculus with constraints on general algebroids, *Journal of Physics A: Mathematical and Theoretical*, vol. 41, no. 17, p. 175204, doi: 10.1088/1751-8113/41/17/175204